Symmetry Point Groups

☞ The complete collection of symmetry operations (*not* symmetry elements) satisfies the requirements of a mathematical group.

✓ The symmetry operations are the *elements* of a group.

✓ The total number of symmetry operations comprising the group is the *order of the group*, *h*.

☛ The group formed by the operations of CBr_2Cl_2 is named C_{2v}.

C_{2v}	E	C_2	σ_v	σ_v'
E	E	C_2	σ_v	σ_v'
C_2	C_2	E	σ_v'	σ_v
σ_v	σ_v	σ_v'	E	C_2
σ_v'	σ_v'	σ_v	C_2	E

Group Requirements

① **Closure** If A and B are in the group G, and $AB = X$, then X is also in G.

✓ All groups have a self-contained multiplication table, whose products are members of the group.

② **Identity** In any group G, there is an element E, such that

$$EX = XE = X$$

✓ The symmetry operation of identity is this group element.

③ **Associativity** If A, B, C, and X are in G, then

$$C(BA) = X = (CB)A$$

✓ But commutation is not general (e.g., $S_4\sigma_v \neq \sigma_v S_4$).

✓ Groups in which all elements *do* commute are called **Abelian** (e.g., C_{2v}).

④ **Reciprocality** In any group G, every element A has an inverse A^{-1}, such that $AA^{-1} = A^{-1}A = E$.

✓ An element may be its own inverse (e.g., all operations of C_{2v}).

Subgroups

☞ Within all groups (except the trivial asymmetric group, $C_1 = \{E\}$), there are smaller collections of elements, called **subgroups**, which also obey the criteria for a group.

- ☛ The order of any subgroup, g, relative to the order of its parent group, h, must be

$$h/g = n \qquad\qquad n = 1, 2, \ldots$$

  - ✓ The order of a subgroup, g, must be an integer divisor of the order of its parent group, h.
  - ✓ More than one subgroup with a particular order g may exist.
  - ✓ Not every allowed value of g is always represented among a group's subgroups.

Subgroups of C_{2v} ($h = 4$)
Possible subgroup orders $g = 1, 2$

Group Label	Operations (Group Elements)	g
C_1	E	1
C_2	E, C_2	2
C_s	E, σ_v or E, σ_v'	2